Appendix E
Discrete Soil Models

E-1. Introduction

This appendix provides a brief description of discrete soil models for a rigid disk on homogeneous half-space, an embedded cylinder, an embedded prism, and a strip supported on a homogeneous half-space. For a more detailed description and formulation refer to Wolf (1988).

E-2. Rigid Disk Supported on Half-Space

A rigid disk supported on a homogeneous half-space undergoes one horizontal and vertical translations and rocking and torsional rotations. A one-dimensional discrete model can approximate the soil medium for each of these four degrees of freedom. The sketch in Table E-1 shows one such model for the vertical component of ground motion. The model is made of mass M_0 attached to the rigid support by spring K and damper C_0 and mass M_1 connected to Node 0 through damper C_1. Node 0, which connects the foundation to the structure, is defined at the center of the disk. The static stiffness K and dimensionless damper and mass coefficients needed to determine M_0, M_1, C_0, and C_1 are given in Table E-1. Note that for some component of motion certain coefficients will be missing. For example only mass M_0 and spring K define the horizontal component of motion, while all coefficients are required to define the vertical component of motion.

E-3. Embedded Cylindrical Foundation

The static stiffness and dimensionless coefficient of a discrete model of massless rigid cylindrical foundation embedded in an elastic homogeneous half-space are summarized in Table E-2. Horizontal and vertical translations and rocking and rotational rotations define the motions of the rigid embedded cylinder, as shown in the sketch in Table E-2. For an embedded foundation, coupling between the horizontal and rocking motions is not negligible and should be considered by connecting the horizontal spring and damper at some distance above the base of the cylinder. These distances for the horizontal spring f_k and horizontal damper f_c are given by:

$$f_k = 0.25e \qquad \text{(E-1)}$$

$$f_c = 0.32e + 0.03e\left(\frac{e}{a}\right)^2 \qquad \text{(E-2)}$$

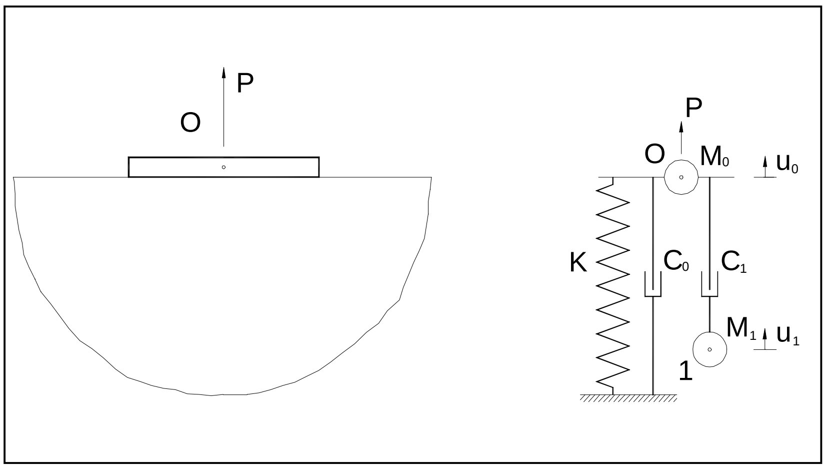
Note that the coefficients given in Table E-2 are available only for $\nu = 0.25$. Also note that by setting $e = 0$, coefficients for the disk on the surface for $\nu = 0.25$ can be obtained from Table E-2.

E-4. Embedded Rectangular Foundation

The static and dimensionless coefficients for an embedded rigid rectangular foundation are given in Table E-3. The foundation is $2l$ long and $2b$ wide ($1 \geq b$) and the embedment expressed as e (sketch in Table E-3). Defined with respect to the center of the basemat, motions of the foundation include vertical and two horizontal translations, two rocking rotations with respect to horizontal axes, and torsional rotation. The properties of the homogeneous half-space and motion notations are also summarized in this sketch. The coupling terms between rocking and horizontal motions are specified as:

Table E-1
Static Stiffness and Dimensionless Coefficients of Discrete Model for Disk Foundation

	Static Stiffness *K*	Dimensionless Coefficients			
		Dampers		Masses	
		γ_0	γ_1	μ_0	μ_1
Horizontal	$\dfrac{8Ga}{2-\nu}$	$0.78-0.4\nu$	—	—	—
Vertical	$\dfrac{4Ga}{1-\nu}$	0.8	$0.34-4.3\nu^4$	$\nu<1/3$: 0; $\nu>1/3$: $0.9(\nu-1/3)$	$0.4-4\nu^4$
Rocking	$\dfrac{8Ga^3}{3(1-\nu)}$	—	$0.42-0.3\nu^2$	$\nu<1/3$: 0; $\nu>1/3$: $0.16(\nu-1/3)$	$0.34-0.2\nu^2$
Torsion	$\dfrac{16Ga^3}{3}$	—	0.29	—	0.2



Basic discrete model

Dimensionless frequency $a_0 = \dfrac{\omega a}{c_s}$

Damping Coefficient $C_0 = \dfrac{a}{c_s} K \gamma_0$

Damping Coefficient $C_1 = \dfrac{a}{c_s} K \gamma_1$

Mass Associated with Node 0 $M_0 = \dfrac{a^2}{c_s^2} K \mu_0$

Mass Associated with Node 1 $M_1 = \dfrac{a^2}{c_s^2} K \mu_1$

Half-space shear modulus *G*
Half-space Poisson's ratio *v*
Half-space Shear Velocity c_s
Disk radius *a*

$$K_{hxry} = \frac{e}{3} K_{hx} \tag{E-3}$$

$$K_{hyrx} = \frac{e}{3} K_{hy} \tag{E-4}$$

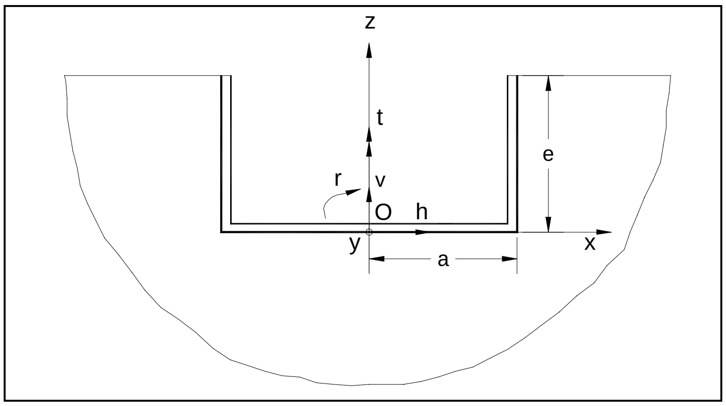
which indicate that the horizontal springs should be connected a distance of $f_{Kx} = f_{Ky} = e/3$ above the base basemat. For simplicity and lack of data, the spring eccentricity can also be used for the horizontal dampers. It should be noted that coefficients of the discrete model in Table E-3 are $\nu = 1/3$. Using these coefficients, masses M_0 and M_1 and dampers C_0 and C_1 are determined from expressions given in the sketch in Table E-1 with rectangular width b replacing the disk radius a. Note that by setting $e = 0$, a rectangular foundation at the ground surface can be treated as a special case of this problem.

E-5. Strip Foundation

The static and dimensionless coefficients of a discrete model of a rigid strip supported on a homogeneous half-plane are provided in Table E-4. The strip is *2b* wide and the half-plane is characterized by *G, v,* and ρ. The

Table E-2
Static Stiffness and Dimensionless Coefficients of Discrete Model for Embedded Cylindrical Foundation

	Static Stiffness K	Dimensionless Coefficients for ν = 0.25			
		Dampers		Masses	
		γ_0	γ_1	μ_0	μ_1
Horizontal	$\frac{8Ga}{2-\nu}\left(1+\frac{e}{a}\right)$	$0.68+0.57\sqrt{e/a}$	—	—	—
Vertical	$\frac{4Ga}{1-\nu}\left(1+0.54\frac{e}{a}\right)$	$0.8+0.35\frac{e}{a}$	$0.32-0.01\left(\frac{e}{a}\right)^4$	—	0.38
Rocking	$\frac{8Ga^3}{3(1-\nu)}\left[1+2.3\frac{e}{a}+0.58\left(\frac{e}{a}\right)^3\right]$	$0.16\frac{e}{a}$	$0.40+0.03\left(\frac{e}{a}\right)^2$	—	$0.33+0.1\left(\frac{e}{a}\right)^2$
Torsion	$\frac{16Ga^3}{3}\left(1+2.67\frac{e}{a}\right)$	—	$0.29+0.09\sqrt{e/a}$	—	$0.20+0.25\sqrt{e/a}$



h = horizontal
v = vertical
r = rocking
t = torsion
a = radius

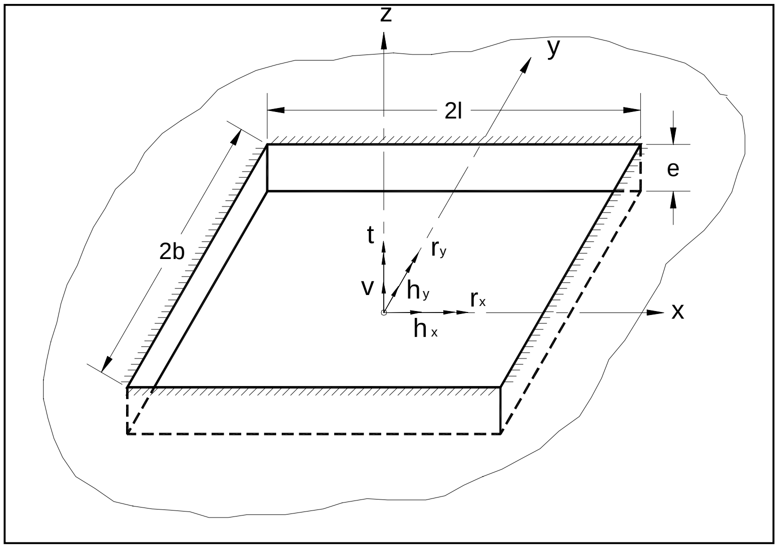
Embedded cylindrical foundation

Note: The lack of reliable data for other Poisson's ratios does not allow the dimensionless coefficients to be specified for as a function of ν.

three motions of the strip include one horizontal, vertical, and rocking. Knowing the coefficients given in Table E-4, masses M_0 and M_1 and dampers C_0 and C_1 are obtained from expressions in the sketch in Table E-1 by substituting b for the disk radius a.

Table E-3
Static Stiffness and Dimensionless Coefficients of Discrete Model for Rectangular Foundation

	Dimensionless Coefficients for $\nu = 1/3$			
	Dampers		Masses	
	γ_0	γ_1	μ_0	μ_1
Horizontal h_x h_y	$0.75+0.2\left(\frac{l}{b}-1\right)$	—	—	—
Vertical	$0.9+0.4\left(\frac{l}{b}-1\right)^{2/3}$	0.3	—	0.14
Rocking r_x	—	0.45	—	0.34
r_y	—	$0.45+0.23\left(\frac{l}{b}-1\right)$		$0.34+0.55\left(\frac{l}{b}-1\right)$
Torsion	—	$0.35+0.12\left(\frac{l}{b}-1\right)$	—	$0.28+0.63\left(\frac{l}{b}-1\right)$
Horizontal translation in x-direction		$K_{hx}=\frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65}+2.4\right]\left[1+\left(0.33+\frac{1.34}{1+\frac{1}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$		
Horizontal translation in y-direction		$K_{hy}=\frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65}+0.8\,\frac{l}{b}+1.6\right]\left[1+\left(0.33+\frac{1.34}{1+\frac{1}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$		
Vertical		$K_v=\frac{Gb}{1-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75}+1.6\right]\left[1+\left(0.25+\frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$		
Rocking about x-axis		$K_{rx}=\frac{Gb^3}{1-\nu}\left(3.2\,\frac{l}{b}+0.8\right)\left[1+\frac{e}{b}+\frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^2\right]$		
Rocking about y-axis		$K_{ry}=\frac{Gb^3}{1-\nu}\left(3.73\,\frac{l}{b}+0.27\right)\left[1+\frac{e}{b}+\frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^2\right]$		
Torsion		$K_t=Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45}+4.06\right]\left[1+\left(1.3+\frac{1.32b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right]$		



Half-space shear modulus	G
Half-space Poisson's ratio	v
Half-space mass density	ρ
Foundation length (1 ≥ b)	$2l$
Foundation width	$2b$
Foundation embedment	e
Horizontal translation in x-dir	h_x
Horizontal translation in y-dir	h_y
Vertical translation	v
Rocking about x-axis	r_x
Rocking about y-axis	r_y
Torsion about z-axis	t

Embedded rectangular foundation

Table E-4
Spring Coefficient and Dimensionless Coefficients of Discrete Model for Strip Foundation (width of strip is 2b)

		Dimensionless Coefficients				
		Dampers			Masses	
	Spring Coefficient *K*	γ_0	γ_1		μ_0	μ_1
Horizontal	$G(1+5v^2)$	$2-2.2v$	—		—	—
Vertical	$G(1+4v^2)$	$3.5-2v$	—	$v<1/3$	0	—
				$v>1/3$	$4.5(v-1/3)$	
Rocking	$Gb^2(1.8+5.2v^2)$	$0.14-0.24v^2$	0.4	$v<1/3$	0	0.3
				$v>1/3$	$0.25(v-1/3)$	